Engineering Notes

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Sensing Relative Attitudes for Automatic Docking

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Introduction

SENSING relative attitudes is a major problem in docking unmanned spacecraft. This method of finding the relative attitudes of two spacecraft measures positions of markers (in this case, cornercube reflectors) on the target vehicle, relative to a sensing device (in this case, a scanning laser radar), on the chase vehicle. This information is then used to calculate relative attitude.

Much work has been done to calculate attitudes of spacecraft. Attitude has been measured by finding stars, including the sun, and by finding the center of mass of the Earth. The strength of a radio signal has been used to measure the angle to the Earth.[1,2] This work is not useful in docking spacecraft when one of the vehicles cannot measure its attitude. The scanning laser radar and cornercube reflectors were expected to be used in this situation, but the scanning laser radar measures the distance to the markers less accurately than the angles of azimuth and elevation, and at short distances the error is absolute rather than relative. The relative error becomes unbounded. A small distance error gives a very large angular error. A statistical approach must be taken so errors can be smoothed by reading several markers. An algorithm for calculating relative attitude from the measured positions of the markers is presented here. The accuracy of the algorithm is controlled by the number and placement of the markers. A theoretical prediction of the accuracy is confirmed by computer simulations. The accuracy increases with the square root of the number of markers.

Overview of the Algorithm

The axis on which the vehicles dock, or longitudinal axis, is the z axis; the z axis of the target is calculated as the normal to the plane of best fit markers. The x and y axes are transverse. Direction cosines of a line lying in the plane of the markers give the y axis of the target vehicle. The x axis of the target is calculated as the cross product of the y axis of the target with its z axis. These axes must line up when the vehicles dock. (See Fig. 1).

Coordinates

The calculations are done in centered differences. In rectangular coordinates of the chase vehicle, the measured positions of the markers are:

$$(x_1,y_1,z_1),(x_2,y_2,z_2),\ldots,(x_k,y_k,z_k)$$

if there are k markers. Let

$$\bar{x}=(x_1+\ldots+x_k)/k,\quad \bar{y}=(y_1+\ldots+y_k)/k,$$

$$\bar{z}=(z_1+\ldots+z_k)/k$$

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and let the centered differences be

$$X=(x_1-\bar{x},\ldots,x_k-\bar{x}),\quad Y=(y_1-\bar{y},\ldots,y_k-\bar{y}),$$

$$Z=(z_1-\bar{z},\ldots,z_k-\bar{z})$$

Plane of Best Fit

Engineering considerations dictate that the markers lie in a plane with the longitudinal axis normal to it. A and B are found by the method of least squares so that the plane of best fit is given by

$$z=Ax+By$$

The direction cosines of the z axis of the target are

$$(-A,-B,1)/(A^2+B^2+1)^{1/2}$$

z is treated as a function of x and y since the change in z is small relative to that in x and y. When the vehicles are in line to dock, all the z measurements will be the same except for error. The mean square distance to the plane of best fit is

$$S=\sum_{j\le k}[z_j-(Ax_j+By_j)]^2$$

To minimize S, partials with respect to A and B are taken and set to 0, giving the following system of equations:

$$(X,X)A+(X,Y)B=(X,Z)$$

$$(X,Y)A+(Y,Y)B=(Y,Z) \qquad (1)$$

where (X,X), (Y,Y), (X,Y), (X,Z), and (Y,Z) denote the usual inner product in R^k. A and B are calculated by Cramer's Rule.

The Line of Best Fit in Three Dimensions

An arrangement of markers that lies symmetric to its center will determine only the longitudinal axis of the spacecraft. In

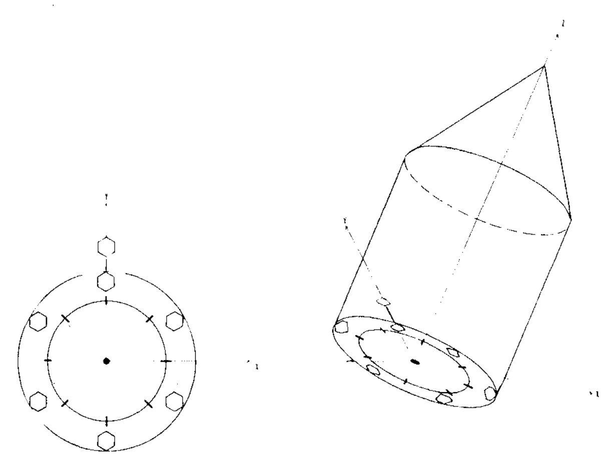

Fig. 1 A spacecraft, or target vehicle, seen from the negative z axis and in perspective. The markers are represented by hexagons. The artist's conception violates right-hand rule.

order to find the transverse axes some asymmetry is necessary. An asymmetric marker is introduced on the y axis outside the circle. The positions of markers which lie opposite each other with respect to the y axis are averaged to locate points which should lie on the y axis. A line in three dimensions is now fitted to these points. Since these points are presumably located on the y axis, both x and z are treated as functions of y. The variation in z will be small if the z axes of the vehicles are almost in line. Averaging points which are opposite over the y axis will give values of x highly correlated with the values of y. If the centered differences of the calculated points are:

$$(\tilde{X}_1, \tilde{Y}_1, \tilde{Z}_1), \ldots, (\tilde{X}_J, \tilde{Y}_J, \tilde{Z}_J)$$

the method of least squares finds m to best fit the points $(\tilde{X}_j, \tilde{Y}_j)$ to the equation

$$x = my$$

M is found by the same method to fit the points $(\tilde{Y}_j, \tilde{Z}_j)$ to

$$z = My$$

Both equations describe planes in three dimensions. The y axis should lie in both and therefore be orthogonal to both normals, $(1, -M, 0)$ and $(0, -m, 1)$. Normalizing their cross product will give the direction cosines of the y axis:

$$(m, 1, M)/(m^2 + 1 + M^2)^{1/2}$$

if the y coordinate of the asymmetric marker is positive; otherwise, the y axis points in the negative direction. If $\tilde{X}$, $\tilde{Y}$, and $\tilde{Z}$ are the centered differences of the calculated points,

$$m = (\tilde{X}, \tilde{Y})/(\tilde{Y}, \tilde{Y}) \qquad \text{and} \qquad M = (\tilde{Z}, \tilde{Y})/(\tilde{Y}, \tilde{Y})$$

Calculations

The positions of the markers are measured as (R, θ, ψ) where

$$x = R\cos\theta\cos\psi, \quad y = R\cos\theta\sin\psi, \quad z = R\sin\theta$$

If the errors in x, y, and z are dx, dy, and dz, respectively, and the errors in R, θ, and ψ are dR, dθ, and dψ, respectively, and $S^2 = x^2 + y^2$, then

$$\mathrm{d}x = x\mathrm{d}R/R - y\mathrm{d}\psi - (x/S)z\mathrm{d}\theta$$

$$\mathrm{d}y = y\mathrm{d}R/R + x\mathrm{d}\psi - (y/S)z\mathrm{d}\theta$$

$$\mathrm{d}z = z\mathrm{d}R/R + S\mathrm{d}\theta \tag{2}$$

Errors in the measured positions of the markers were simulated by generating a normally distributed error for each of the measurements in spherical coordinates and using Eq. (2) to generate dx, dy, and dz.

The markers were assumed to be evenly spaced about a circle with one marker off the circle to distinguish the y axis. The following data were input: the angles of rotation of the axes, the distance between the spacecraft, the radius of the circle of markers, the number of markers, the variance of the errors in measuring the positions of the markers, and the number of readings of the position of each marker. For each marker, a true position was calculated, errors generated, and a measured position recorded.

The algorithm for the plane of best fit was used to calculate the z axis of the target vehicle. Points lying across the y axis were averaged to get points calculated to lie on the y axis. The line-of-best-fit algorithm was applied to find the y axis. The x axis was calculated to be the cross product of the calculated y and z axes.

Predicted Error

Let

$$F = (X,X)(Y,Y) - (X,Y)^2$$

$$G = (X,Z)(Y,Y) - (Y,Z)(X,Y),$$

and dA, dF, and dG be the errors in A, F, and G. Then Eq. (1) implies $A = G/F$ and

$$\mathrm{d}A = (\mathrm{d}G - A\mathrm{d}F)/F$$

The larger F is relative to the inner products that appear in Eq. (1), the more accurately A will be known. Similar statements hold for B. To make F large, (X, Y) must be small relative to (X,X) and (Y,Y). This implies that both $|X|$ and $|Y|$ should be as large as possible, while (X, Y), the covariance of X and Y, should vanish or be small. For $|X|$ and $|Y|$ to be large, the markers should be set far away from their center. For (X, Y), the covariance of X and Y, to be small the markers should be in a configuration that does not approximate a straight line. If the markers are arranged so that they lie symmetric with respect to either the x or y axis, covariance will vanish except for error in the readings when the plane of the markers is parallel to the xy plane of the chase vehicle. Since the covariance is continuous, (X, Y) will be small when the plane of the markers is only slightly tilted from that plane. This is, in fact, the most delicate situation for docking. This increase in accuracy of the algorithm when the spacecraft are almost in line may somewhat offset the large relative error in the distance measurements when the spacecraft are close.

Error Calculations

The angular errors were calculated as the inverse cosine of the dot product of the true and calculated z axes. The z axis was chosen because, while this is not the case in our calculations, it is easy to imagine a situation where the vehicles need only match the z axes to dock. In addition, the z-axis errors were largest except at high angles. This calculation is unstable when the angle between the vectors is small since a small change in the cosine will give a large change in the angle.[3]

The standard deviations of the angular errors were calculated after ten runs for each fixed set of conditions. The accuracy of the calculation is measured as the standard deviation of the error. A drop in this standard deviation is interpreted as an increase in accuracy.[4,5]

Results

Standard deivations for the error were calculated at distances of 200, 100, 50, 25, 12.5, and 6.25 ft for each arrangement of the markers at various angles of approach. The variations tested included 6 or 12 markers spaced about

Table 1 Results of a typical set of runs

7 Reflectors			13 Reflectors		
1 ft radius	2 ft radius	3 ft radius	1 ft radius	2 ft radius	3 ft radius
3.76 ± 0.72	1.65 ± 0.08	1.11 ± 0.10	2.49 ± 0.28	1.21 ± 0.13	0.86 ± 0.04

Table 2 Ratio of standard deviations at different distance

	7 Reflectors	13 Reflectors	Predicted
σ_1/σ_2	2.27	2.06	2.0
σ_2/σ_3	1.49	1.41	1.5
σ_1/σ_3	3.39	2.89	3.0

circles of radius 1, 2, or 3 ft with an additional asymmetric marker 1 ft beyond the circle on the y axis.

There was little change in error with the simulated change in attitude. The values of the error were calculated at each distance and plotted against the log of the distance. The changes over the distance from 6.25 to 200 ft were so small as to not affect the standard error by as much as 0.2 deg. Results from a typical set of runs are shown in Table 1.

The ratios between the standard deviations of the errors were predicted to be inversely proportional to the square roots of the numbers of markers, roughly 1.36. The simulation yielded 1.58, 1.38, and 1.31 for 1, 2, and 3 ft, respectively. The ratios of standard deviations at different distances were compared in Table 2 where σ_j represents the standard deviation for a circle of radius j.

To explore the effect of an increase in the accuracy of the scanning laser radar, in some of the runs it was assumed that each marker was read three times and the result averaged to get a more accurate position rading. The predicted ratio between the standard errors was 0.577. When run with 13 reflectors and a radius of 3 ft, this gave a standard error of 0.49 ± 0.04 deg. The ratio of the corresponding standard errors was 0.568.

Conclusions

A method for determining the relative attitude of two docking unmanned spacecraft has been developed. The method gives predictable and controllable accuracy. If implemented with scanning laser radar and cornercube reflectors or a congruent system, the markers should be arranged as far from their center as possible. This will increase the radius of the circle which will improve the accuracy of the readings in two ways: 1) the coordinates are known more accurately, and just as important, 2) F is increased along with $|Y|$. Increasing the denominators of the calculated quantities directly increases their accuracy. To get F large, it is also necessary that (X, Y) be small, so the markers, except for the asymmetric marker, should be set symmetric to both axes.

There was a great increase in accuracy when the radius, R, of the circle of markers was increased. This was true at all distances and all rotations tested. The standard deviation of the errors appears to decrease as $1/R$, as expected.

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A Core Software Concept for Integrated Control

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Introduction

INTEGRATED control[1,2] makes use of digital computer technology, digital data communications, and software engineering to integrate navigation, guidance, communications, energy management, stores management, and for those missions requiring weapon delivery, target acquisition/identification and weapon delivery functions with aircraft control functions (outer and inner loops). While these same functions have been performed since the beginning of military aviation, it is only recently, with the progression from no computers to analog, and now to digital computers, that it has become possible to integrate the functions efficiently and reduce the duplication of sensors and processing.

Digital systems, such as the integrated control system, utilize a digital communication architecture made up of computers, digital data transfer channels, and the associated sensors and controllers required to implement the control process. The management of this network of computers, interconnected via the digital data transfer channels, as well as the other integrated control functions, comprises the primary system management function. The system management function may be implemented with both hardware and software. The software must implement the system management operating system (executive software for control of communication protocol which includes software-implemented fault detection and recovery) and interface with the local executive in each processor. The system management function performance is critical to the successful implementation of integrated control.

System architectures, such as the triple redundant mechanization with self-test and in-line monitoring, are representative of concepts for implementing a fault-tolerant system management function in conjunction with the flight control function. The use of majority voting exemplifies present concepts for avoidance of hardware and software faults.

A particular function, such as the control function, may be done automatically, manually, or by a combination of automatic and manual processes. The combination of processes for a particular function shall be defined as specific subfunctions. Each subfunction may be performed in a variety of modes.

A mode is defined as a specific set of measurements (inputs) and appropriate algorithms which are processed to provide desired outputs. These measurements may be provided by specific combinations of sensor hardware and software, or may make use of optimal estimation theory (such as a Kalman filter) to compute or estimate the vehicle state based on input from sensors which may provide redundant measurements or information. The specific mode that is utilized in performing a function is dependent upon a hierarchy of modes established by the system designer for automatic subfunctions and the crew selection of modes using manual subfunctions, as well as the status of the system where status is defined as the hard-

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